

Perturbative QCD analysis of b -hadron lifetimes

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PACS numbers: 13.20.He, 12.38.Bx, 12.38.Cy, 14.40.Nd

Abstract

We develop perturbative QCD factorization theorems for inclusive b -hadron decays, in which radiative corrections characterized by the hadronic scale, the b -hadron mass, and the W boson mass are absorbed into a heavy hadron distribution function, a hard b quark decay amplitude, and a “harder” function, respectively. Double logarithmic corrections associated with a light energetic final-state quark, which appear at kinematic end points, are absorbed into a jet function. Various large logarithms contained in the above functions are summed to all orders, leading to the evolution factors among the three characteristic scales. The heavy hadron distribution function is identical to the one constructed in the framework of heavy quark effective theory. It is shown that hadron kinematics must be employed in factorization theorems, and that perturbative contributions, depending on hadron kinematics, distinguish the lifetimes of the b -hadrons B_d , B_s and Λ_b . Assuming the same heavy-quark-effective-theory parameters λ_1 for these hadrons, we predict the lifetimes $\tau(B_d) = 1.56$ ps, $\tau(B_s) = 1.46$ ps and $\tau(\Lambda_b) = 1.22$ ps. We also predict the B_u meson lifetime $\tau(B_u) = 1.62$ ps by varying the B meson distribution function slightly. All the above results are consistent with experimental data.

I. INTRODUCTION

One of the puzzles in heavy hadron decays is the explanation of the low lifetime ratio $\tau(\Lambda_b)/\tau(B_d) = 0.79 \pm 0.06$ [1]. Inclusive heavy hadron decays involve both nonperturbative and perturbative corrections to the tree-level b quark decay amplitudes. In the framework of heavy quark effective theory (HQET) [2], nonperturbative corrections are expanded in powers of $1/M_b$, M_b being the b quark mass, with the coefficient of each power proportional to a hadronic matrix element of local operators, and perturbative corrections are evaluated order by order at the quark level. The HQET prediction for the ratio $\tau(\Lambda_b)/\tau(B_d)$ to $O(1/M_b^2)$ is about 0.99 [3]. When including the $O(1/M_b^3)$ corrections, the ratio depends on six unknown parameters, and reduces to around 0.97 for various model estimates, which is still far beyond the experimental data. On the other hand, a phenomenological ansatz was proposed, in which the overall M_b^5 factor in front of nonleptonic decay widths is replaced by the corresponding hadron mass M_H^5 [4]. This ansatz provides a solution to the puzzle, since $(M_B/M_{\Lambda_b})^5 = 0.73$, M_B and M_{Λ_b} being the B meson mass and the Λ_b baryon mass, respectively, is close to the observed ratio. As pointed out in [5], the same replacement also explains the absolute B meson decay rate, while the HQET prediction using the expansion parameter M_b accounts for only 80% of the decay rate.

An alternative approach to inclusive heavy hadron decays is the perturbative QCD (PQCD) factorization theorem [6]. In this formalism radiative corrections to b quark decays are absorbed into different factors in factorization formulas for decay widths according to their characteristic scales. The soft corrections characterized by the hadronic scale of order Λ_{QCD} are factorized into a universal heavy hadron distribution function. The corrections characterized by the heavy hadron mass and by the W boson mass are factorized into a hard b quark decay amplitude and a “harder” function, respectively. The heavy hadron distribution function provides a summation over the initial states of the b quark, such that the virtual and real soft gluon corrections cancel exactly. The introduction of a distribution function indicates that a residual momentum

of the b quark, which arises as an effect from the light degrees of freedom in the heavy hadron, is allowed. The inclusion of the light degrees of freedom then demands the use of the heavy hadron kinematics, whose difference from the b quark kinematics sets a constraint on the magnitude of the residual momentum.

The soft gluons are collected into the distribution function by employing an eikonal approximation, under which a b quark propagator is simplified into the propagator associated with the large component of the rescaled b quark field in HQET. This observation implies the identity of the heavy hadron distribution function to the one constructed in the HQET framework [7]. Therefore, nonperturbative corrections also start from $O(1/M_b^2)$ and the quark-hadron duality is respected in the PQCD formalism, the same as in HQET. However, as transforming the b quark kinematics to the heavy hadron kinematics through $M_b = M_H - \bar{\Lambda}$, the $O(\bar{\Lambda}/M_H)$ correction occurs [7]. It has been explicitly demonstrated, taking the $B \rightarrow X_u l \bar{\nu}$ as an example [6], that the $O(1/M_B)$ correction from the B meson distribution function cancels that from the phase space factor M_B^5 .

As to perturbative corrections, various large logarithms contained in b quark decays are summed to all orders in the PQCD approach. The results are the renormalization-group (RG) evolutions among the three characteristic scales, such that the factorization formulas are independent of the renormalization scale μ . Double logarithms associated with light energetic final-state quarks are organized by the Collins-Soper resummation technique [8], leading to a Sudakov factor which smears end-point singularities and improves the applicability of PQCD. These perturbative factors, depending on kinematics of the initial heavy hadrons and of final states, differ among various decay modes of a hadron and among various hadrons. This is natural, because different decay modes should involve different energy releases. In the HQET approach perturbative corrections are evaluated to finite orders explicitly at the quark level with the renormalization scale μ set to a common value for all decay modes. It is then not a surprise that HQET predicts the lifetime ratio $\tau(\Lambda_b)/\tau(B_d) \approx 0.99$, since nonperturbative corrections, supposed to distinguish the decays of different b -hadrons, are suppressed by $1/M_b^2$, and perturbative corrections to b quark decays are the same.

The B meson distribution function has been extracted from the photon energy spectrum of the $B \rightarrow X_s \gamma$ decay [9], which provides the information of the b quark mass M_b and of the HQET parameter λ_1 for the B_d meson. Based on this information, we propose reasonable B_s meson and Λ_b baryon distribution functions by assuming that they correspond to the same λ_1 . Convoluting these distribution functions with the perturbative factors, we predict the correct lifetimes of the b -hadrons: $\tau(B_d) = 1.56$ ps, $\tau(B_s) = 1.46$ ps and $\tau(\Lambda_b) = 1.22$ ps, and thus the correct ratios $\tau(B_s)/\tau(B_d) = 0.94$ and $\tau(\Lambda_b)/\tau(B_d) = 0.78$. By varying the B meson distribution function slightly, we obtain the B_u meson lifetime $\tau(B_u) = 1.62$ ps, and the ratio $\tau(B_u)/\tau(B_d) = 1.04$. It will be shown that perturbative contributions play an essential role in the explanation of the b -hadron lifetimes, and that our predictions are insensitive to the variation of the distribution functions, as expected from the quark-hadron duality. Our results of the semileptonic branching ratio $B_{\text{SL}} = B(B \rightarrow X l \bar{\nu}) = 10.16\%$ and of the average charm yield $\langle n_c \rangle = 1.17$ per B decay are also consistent with experimental data. We present the semileptonic branching ratios and the charm yields in B_s meson and Λ_b baryon decays as well, which can be tested in future experiments.

In Sec. II the factorization theorem for semileptonic b -hadron decays is constructed. The equivalence of the B meson distribution function to that derived in the HQET framework is demonstrated in Sec. III. The three-scale factorization theorem for nonleptonic decays is developed in Sec. IV. Various logarithmic corrections are summed to all orders using the resummation technique and RG equations in Sec. V. In Sec. VI we present the factorization formulas for the b -hadron decay widths and their numerical analysis. Section VII is the conclusion. The appendix contains the detailed derivation of the allowed phase space for heavy hadron decays.

II. FACTORIZATION OF SEMILEPTONIC DECAYS

We start with the factorization of the simplest case, the semileptonic decays $B \rightarrow X_c l \bar{\nu}$, which correspond to the $b \rightarrow c l \bar{\nu}$ decays at the quark level. These decays involve only the B meson distribution function and the hard b quark decay amplitudes due to the lack of the characteristic scale of the W boson mass. Nonperturbative dynamics is reflected by infrared poles in radiative corrections to quark-level amplitudes in perturbation theory. According to PQCD factorization theorems, these poles are absorbed into a hadron distribution function, which must be derived by nonperturbative methods or extracted from experimental data. A distribution function, being universal, is determined once for all, and then employed to make predictions for other processes containing the same hadron. In this section we demonstrate how to isolate infrared poles from radiative corrections and factorize them into the B meson distribution function.

Consider the one-loop corrections to the $b \rightarrow cl\bar{\nu}$ decays shown in Fig. 1. Because both the b and c quarks are massive, there are no collinear (mass) divergences, and we concentrate only on soft divergences from vanishing loop momenta. The self-energy correction to the b quark in Fig. 1(a) is written as

$$\Sigma^{(a)} u_b = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{1}{l^2} \frac{\not{p}_b + M_b}{p_b^2 - M_b^2} \gamma_\mu \frac{\not{p}_b - \not{l} + M_b}{(p_b - l)^2 - M_b^2} \gamma^\mu u_b \delta(p_c^2 - M_c^2), \quad (1)$$

with $C_F = 4/3$ a color factor, p_b and u_b the b quark momentum and spinor, respectively, and $p_c = p_b - q$ and M_c the c quark momentum and mass, respectively, q being the lepton pair momentum. The δ -function from the final-state cut on the outgoing c quark is included for the discussion of the soft cancellation between virtual and real corrections below. The b quark propagator $(\not{p}_b + M_b)/(p_b^2 - M_b^2)$ after the self-energy correction helps the extraction of the soft pole in Eq. (1).

The soft divergence of $\Sigma^{(a)}$ is isolated by the eikonal ($l \rightarrow 0$) approximation,

$$\Sigma_{\text{soft}}^{(a)} u_b = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{1}{l^2} \frac{\not{p}_b + M_b}{p_b^2 - M_b^2} \gamma_\mu \frac{2p_b^\mu}{-2p_b \cdot l + p_b^2 - M_b^2} u_b \delta(p_c^2 - M_c^2), \quad (2)$$

where the term \not{l} in the numerator and l^2 in the denominator of the b quark propagator have been neglected, and the equation of motion $(\not{p}_b - M_b)u_b = 0$ has been applied to obtain the factor $2p_b^\mu$. We make the following expansion because of the on-shell condition $p_b^2 - M_b^2 \rightarrow 0$:

$$\frac{1}{-2p_b \cdot l + p_b^2 - M_b^2} = -\frac{1}{2p_b \cdot l} - \frac{p_b^2 - M_b^2}{4(p_b \cdot l)^2}, \quad (3)$$

where the first term on the right-hand side does not contribute, since it leads to an integrand with an odd power in l . The numerator of the second term removes the on-shell pole of the b quark propagator after the self-energy correction. Employing the equation of motion again, we obtain

$$\Sigma_{\text{soft}}^{(a)} = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{v^2}{l^2 (v \cdot l)^2} \delta(p_c^2 - M_c^2), \quad (4)$$

where $v \equiv p_b/M_b$ is the b quark velocity. Obviously, Eq. (4) does not depend on the spin and mass of the b quark.

The loop integral for Fig. 1(b) with a real gluon attaching the b quarks before and after the final-state cut is written as

$$u_b \Sigma^{(b)} u_b = -g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} u_b \gamma_\mu \frac{\not{p}_b - \not{l} + M_b}{(p_b - l)^2 - M_b^2} \Gamma \frac{\not{p}_b - \not{l} + M_b}{(p_b - l)^2 - M_b^2} \gamma^\mu \times 2\pi \delta(l^2) \delta((p_c - l)^2 - M_c^2), \quad (5)$$

where Γ represents the other irrelevant vertices and propagators. The δ -function is associated with the final-state c quark with the momentum $p_c - l = p_b - q - l$ in the case of real gluon emission. The soft divergence in Eq. (5) is also extracted by applying the eikonal approximation and the equation of motion to the two b quark propagators, leading to

$$\Sigma_{\text{soft}}^{(b)} = -g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{v^2}{(v \cdot l)^2} 2\pi \delta(l^2) \delta((p_c - l)^2 - M_c^2). \quad (6)$$

The above expression indicates that under the eikonal approximation, the b quark propagator is replaced by the eikonal propagator $1/(v \cdot l)$ and the quark-gluon vertex γ^μ is replaced by the eikonal vertex v^μ . These Feynman rules for an eikonal line are exactly the same as those associated with the large component $h_v(x)$ of the rescaled b quark field $b_v(x)$,

$$h_v(x) = \frac{1 + \not{v}}{2} b_v(x), \quad b_v(x) = \exp(iM_b v \cdot x) b(x), \quad (7)$$

that appears in HQET. This observation will become essential, as we demonstrate the identity of the B meson distribution functions constructed in the PQCD factorization theorems and in HQET.

A straightforward calculation gives

$$\Sigma_{\text{soft}}^{(a)} = \frac{\alpha_s}{2\pi} C_F \frac{1}{-\epsilon} \delta(p_c^2 - M_c^2) , \quad (8)$$

$$\Sigma_{\text{soft}}^{(b)} = -\frac{\alpha_s}{\pi} C_F \frac{(\mu^2 p_c^2)^\epsilon}{(p_c^2 - M_c^2)^{1+2\epsilon}} . \quad (9)$$

The ultraviolet pole in $\Sigma_{\text{soft}}^{(a)}$ has been subtracted, $1/(-\epsilon)$ with $\epsilon < 0$ denotes the soft pole, and the other finite terms irrelevant to our discussion have been dropped. The real gluon correction $\Sigma_{\text{soft}}^{(b)}$ in fact contains a soft pole as $p_c^2 \rightarrow M_c^2$. Following the standard factorization of deep inelastic scattering, this pole is extracted through the convolution with a B meson structure function. The introduction of such a structure function implies that the b quark momentum p_b is allowed to vary. A variable p_b is natural, since the light degrees of freedom in the B meson share various amount of meson momentum, such that the b quark is not always at rest.

Define the residual momentum of the b quark as $k = p_b - M_b v$, $v = (v^+, v^-, \mathbf{v}_\perp) = (1, 1, \mathbf{0})/\sqrt{2}$ in the light-cone notation, for which $k = (k^+, 0, \mathbf{0})$ is a convenient parametrization. The B meson structure function $f(k^+)$ then describes the probability of finding a b quark with the residual momentum k^+ inside the B meson. Consequently, the one-loop corrections in Figs. 1(a) and 1(b) are modified into the convolutions of Eqs. (1) and (5) with $f(k^+)$, respectively, where the c quark momentum is replaced by $p_c = M_b v + k - q$. It is legitimate to reexpress the factor $1/(p_c^2 - M_c^2)^{1+2\epsilon}$ in Eq. (9) as

$$\frac{1}{(p_c^2 - M_c^2)^{1+2\epsilon}} = \frac{1}{-2\epsilon} \delta(p_c^2 - M_c^2) + \frac{1}{(p_c^2 - M_c^2)_+} . \quad (10)$$

The second term in the above expression is defined via the convolution with an arbitrary function $F(k^+)$,

$$\int dk^+ \frac{F(k^+)}{(p_c^2 - M_c^2)_+} = \int dk^+ \frac{F(k^+) - F(\bar{k})}{p_c^2 - M_c^2} , \quad (11)$$

in which

$$\bar{k} = -\frac{M_b^2 - 2M_b q^0 + q^2 - M_c^2}{2(M_b v^- - q^-)} , \quad (12)$$

is the value of k^+ determined by the on-shell condition $p_c^2 = M_c^2$. Apparently, Eq. (11) is infrared finite as $k^+ \rightarrow \bar{k}$ ($p_c^2 \rightarrow M_c^2$), and thus the second term in Eq. (10) is negligible. The first term in Eq. (10) leads to

$$\Sigma_{\text{soft}}^{(b)} = -\frac{\alpha_s}{2\pi} C_F \frac{1}{-\epsilon} \delta(p_c^2 - M_c^2) , \quad (13)$$

which cancels $\Sigma_{\text{soft}}^{(a)}$ in Eq. (8).

A similar cancellation occurs between the self-energy correction to the c quark in Fig. 1(c) and its corresponding real gluon correction in Fig. 1(d), when they are convoluted with the B meson structure function. The loop integrals are the same as those in Eqs. (4) and (6) but with the b quark velocity v replaced by the c quark velocity $v_c \equiv p_c/M_c$. Using the eikonal approximation, the soft structure of the vertex correction in Fig. 1(e) with the virtual gluon attaching the b and c quarks is given by

$$\Sigma_{\text{soft}}^{(e)} = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{v \cdot v_c}{l^2 v \cdot l v_c \cdot l} \delta(p_c^2 - M_c^2) . \quad (14)$$

The soft structure extracted from the corresponding real correction in Fig. 1(f) is

$$\Sigma_{\text{soft}}^{(f)} = g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{v \cdot v_c}{v \cdot l v_c \cdot l} 2\pi \delta(l^2) \delta((p_c - l)^2 - M_c^2) . \quad (15)$$

Following the similar reasoning, we show that $\Sigma_{\text{soft}}^{(e)}$ and $\Sigma_{\text{soft}}^{(f)}$ cancel exactly. The above soft cancellation can be generalized to all orders straightforwardly using the eikonal approximation, under which soft gluons detach from the quarks. In conclusion, the exact soft cancellation between virtual and real corrections to b quark decays must be implemented by introducing the B meson structure function. This is consistent with the Kinoshita-Lee-Nauenberg theorem, which states that infrared divergences in radiative corrections to a QCD process cancel when both final and initial states are summed. The B meson structure function $f(k^+)$ simply provides a weighting factor for the summation over the initial b quark states.

The one-loop contributions to the hard b quark decay amplitude H for the semileptonic decays are defined as the difference of the full radiative corrections and their eikonalized versions, that have been absorbed into the B meson structure function. As a demonstration, we perform the factorization of the self-energy correction to the b quark:

$$\begin{aligned}
& (\text{tree diagram}) + \text{Fig. 1(a)} \\
&= (\text{tree diagram}) + \text{Fig. 1(a)} - (\text{tree diagram}) \times \Sigma_{\text{soft}}^{(a)} + (\text{tree diagram}) \times \Sigma_{\text{soft}}^{(a)}, \\
&= [(\text{tree diagram}) + \text{Fig. 1(a)} - (\text{tree diagram}) \times \Sigma_{\text{soft}}^{(a)}] \times (1 + \Sigma_{\text{soft}}^{(a)}) + O(\alpha_s^2). \tag{16}
\end{aligned}$$

The first and second factors in the last line of the above expression belong to the first-order H and $f(k^+)$, respectively. Extending the above procedures to all orders, we derive the factorization formula for the semileptonic decays, which is a convolution of the hard amplitude with the B meson structure function. Because of the soft subtraction $(\text{tree diagram}) \times \Sigma_{\text{soft}}^{(a)}$, H is infrared finite and calculable at the quark level in perturbation theory.

III. MOMENTS OF THE DISTRIBUTION FUNCTION

A formal definition of the B meson structure function $f(k^+)$ should reflect the soft dynamics associated with the B meson. Hence, we replace the b quark line by an eikonal line in the direction v , which collects infinite many soft gluons radiated by the b quark. From Eq. (7), the dependence on the b quark mass M_b is removed by employing the rescaled b quark field b_v . Specifying the large component h_v of b_v , the spin of the b quark remains fixed as radiating gluons, and its dependence also decouples. Therefore, the propagator and the gluon vertex associated with h_v in HQET are exactly the same as the eikonal propagator and vertex, implying that $f(k^+)$ is defined in terms of h_v . The initial and final b quarks have a separation y^- in the minus coordinate, since the b quark momentum varies in the plus direction. At last, to render the definition gauge invariant, we insert a path-ordered exponential $P \exp[i \int_0^{y^-} dt n \cdot A(tn)]$ in between the initial and final b quark fields, with $n = (0, 1, \mathbf{0})$ a vector on the light cone, forming

$$f(k^+) = \int \frac{dy^-}{2\pi} e^{ik^+ y^-} \langle B(v) | \bar{h}_v(0) P \exp \left[-i \int_0^{y^-} dt n \cdot A(tn) \right] h_v(y^-) | B(v) \rangle, \tag{17}$$

where $|B(v)\rangle$ is the initial state of the B meson. The Feynman rules for the path-ordered exponential are an eikonal line in the direction n with the propagator $1/(n \cdot l)$ and the vertex n^μ .

The effects of the variation of the b quark momentum from the mass shell, determined by $f(k^+)$, are nonperturbative. Because the variation is of order Λ_{QCD} , it is appropriate to expand the nonperturbative corrections contained in $f(k^+)$ in terms of its moments [7]:

$$\int dk^+ f(k^+) = \langle B(v) | \bar{h}_v(0) h_v(0) | B(v) \rangle = 1, \tag{18}$$

$$\int dk^+ k^+ f(k^+) = \langle B(v) | \bar{h}_v(0) i D^+ h_v(0) | B(v) \rangle = 0, \tag{19}$$

$$\int dk^+ k^{+2} f(k^+) = \langle B_v | \bar{h}_v(0) (i D^+)^2 h_v(0) | B(v) \rangle \equiv -\lambda_1/6, \tag{20}$$

where Eq. (18) is the normalization of $f(k^+)$, Eq. (19) is the consequence of the equation of motion for h_v , and Eq. (20) defines the HQET parameter λ_1 . Equation (19) implies that the $O(1/M_b)$ nonperturbative correction to the semileptonic decays does not exist, as observed in HQET. That is, the quark-hadron duality, which states the equality of heavy hadron decay widths to heavy quark decay widths up to small difference of $O(1/M_b^2)$ from λ_1 , is respected in the PQCD formalism.

As argued before, the B meson structure function allows a variable b quark momentum, whose source is the effect of the light degrees of freedom in the B meson. This effect demands the consideration of B meson, instead of b quark, decays, and thus the B meson kinematics. Therefore, we transform the structure function $f(k^+)$ into the usual B meson distribution function $f_B(z)$, $z = p_b^+ / P_B^+$, which describes the probability to find a b quark with the plus momentum $z P_B^+$ [7]. The relation between $f(k^+)$ and $f_B(z)$ can be extracted from their moments by employing the variable transformation,

$$k^+ = p_b^+ - M_b v^+ = \frac{z M_B - M_b}{\sqrt{2}}. \quad (21)$$

Equation (18) leads to the normalization of $f_B(z)$,

$$\int_0^1 dz f_B(z) = 1, \quad f_B(z) \equiv \frac{M_B}{\sqrt{2}} f\left(\frac{z M_B - M_b}{\sqrt{2}}\right). \quad (22)$$

Equation (19) gives

$$\int dz (1-z) f_B(z) = \left(1 - \frac{M_b}{M_B}\right) \int dz f_B(z) = \frac{\bar{\Lambda}}{M_B} + O\left(\frac{\Lambda_{\text{QCD}}^2}{M_B^2}\right), \quad (23)$$

into which Eq. (22) has been inserted. Applying a similar manipulation to Eq. (20), we arrive at

$$\int dz (1-z)^2 f_B(z) = \frac{1}{M_B^2} \left(\bar{\Lambda}^2 - \frac{\lambda_1}{3}\right) + O\left(\frac{\Lambda_{\text{QCD}}^3}{M_B^3}\right). \quad (24)$$

The HQET parameters $\bar{\Lambda}$ and λ_1 satisfy the relation

$$\bar{\Lambda} = M_B - M_b + \frac{\lambda_1}{2M_b}. \quad (25)$$

Therefore, the heavy hadron kinematics is introduced into the PQCD formalism rigorously via Eqs. (17)-(24). It is then clear that the PQCD approach is equivalent to HQET in the treatment of nonperturbative corrections, and the moment $\bar{\Lambda}/M_B$ is attributed to the replacement of the b quark mass by the B meson mass. An advantage of the B meson kinematics is that it provides the correct kinematic bounds. Take the lepton energy spectrum as an example. Using the b quark kinematics, the maximal lepton energy $M_b/2$ is smaller than the correct value $M_B/2$.

The B meson distribution function must be determined by means outside the PQCD regime. For its functional form, we propose [6]

$$f_B(z) = N \frac{z(1-z)^2}{[(z-a)^2 + \epsilon z]^2}, \quad (26)$$

where the three parameters N , a and ϵ are obtained from the best fit to experimental data. We have extracted $f_B(z)$ from the photon energy spectrum of the radiative decay $B \rightarrow X_s \gamma$, given by [9]

$$f_B(z) = \frac{0.02647 z(1-z)^2}{[(z-0.95)^2 + 0.0034z]^2}. \quad (27)$$

The maximum of f_B occurs at $z \sim 1$ as expected, since the b quark carries most of the B meson momentum. Substituting the above expression into Eqs. (22), (23) and (24), which relate the three parameters N , a and ϵ to the HQET parameters, we obtain $\bar{\Lambda} = 0.65$ GeV and $\lambda_1 = -0.71$ GeV².

As to the distribution functions for other b -hadrons, we assume the parametrizations of the B_s meson and Λ_b baryon distribution functions, $f_{B_s}(z)$ and $f_{\Lambda_b}(z)$, respectively, the same as Eq. (26). The moments of $f_{B_s(\Lambda_b)}$ need to be treated as free parameters, since they have not been determined yet. The relations of the parameters N , a and ϵ to the first three moments of $f_{B_s(\Lambda_b)}$ are similar to Eqs. (22), (23) and (24):

$$\begin{aligned} \int_0^1 f_{B_s(\Lambda_b)}(z) dz &= 1, \\ \int_0^1 (1-z) f_{B_s(\Lambda_b)}(z) dz &= \frac{\bar{\Lambda}_{B_s(\Lambda_b)}}{M_{B_s(\Lambda_b)}} + O(\Lambda_{\text{QCD}}^2/M_{B_s(\Lambda_b)}^2), \\ \int_0^1 (1-z)^2 f_{B_s(\Lambda_b)}(z) dz &= \frac{1}{M_{B_s(\Lambda_b)}^2} \left(\bar{\Lambda}_{B_s(\Lambda_b)}^2 - \frac{1}{3}\lambda_{1B_s(\Lambda_b)}\right) + O(\Lambda_{\text{QCD}}^3/M_{B_s(\Lambda_b)}^3), \end{aligned} \quad (28)$$

with

$$\begin{aligned}\lambda_{1B_s(\Lambda_b)} &= -6\langle B_s(\Lambda_b) | \bar{b}(iD_\perp)^2 b | B_s(\Lambda_b) \rangle \\ M_{B_s(\Lambda_b)} &= M_b + \bar{\Lambda}_{B_s(\Lambda_b)} - \frac{\lambda_{1B_s(\Lambda_b)}}{2M_b}.\end{aligned}\quad (29)$$

Note that only λ_{1B_s} and $\lambda_{1\Lambda_b}$ are free parameters. Because of the parameters $\bar{\Lambda} = 0.65$ GeV and $\lambda_1 = -0.71$ GeV² and $M_B = 5.279$ GeV, the b quark mass $M_b = 4.551$ GeV is fixed by Eq. (25). Substituting this M_b into Eq. (29) with $M_{B_s} = 5.369$ GeV and $M_{\Lambda_b} = 5.621$ GeV, $\bar{\Lambda}_{B_s}$ and $\bar{\Lambda}_{\Lambda_b}$ will be derived, if λ_{1B_s} and $\lambda_{1\Lambda_b}$ are chosen. Using the values of $\bar{\Lambda}_{B_s(\Lambda_b)}$ and $\lambda_{1B_s(\Lambda_b)}$, combined with the normalization of $f_{B_s(\Lambda_b)}$, we obtain the corresponding parameters N , a and ϵ . In the numerical analysis we shall assume that the HQET parameters λ_1 are the same for all b -hadrons, *i.e.*, $\lambda_{1B_s} = \lambda_{1\Lambda_b} = \lambda_1 = -0.71$ GeV², which correspond to $\bar{\Lambda}_{B_s} = 0.74$ GeV and $\bar{\Lambda}_{\Lambda_b} = 0.99$ GeV. The B_s meson and Λ_b baryon distribution functions are then given by

$$f_{B_s}(z) = \frac{0.02279z(1-z)^2}{[(z-0.93)^2 + 0.0043z]^2}, \quad (30)$$

$$f_{\Lambda_b}(z) = \frac{0.02095z(1-z)^2}{[(z-0.89)^2 + 0.0068z]^2}. \quad (31)$$

For the B_u meson distribution function, the reasoning is a bit different. In fact, we can not distinguish the B_u and B_d meson distribution functions, if f_B is extracted from the photon energy spectrum of the $B \rightarrow X_s \gamma$ decay, which acquires contributions from both mesons [10]. To explain the B_u meson lifetime in Sec. VI, we shall vary f_B slightly:

$$f_{B_u}(z) = \frac{0.03358z(1-z)^2}{[(z-0.96)^2 + 0.0034z]^2}, \quad (32)$$

which corresponds to $\bar{\Lambda}_{B_u} = 0.65$ GeV and $\lambda_{1B_u} = -0.81$ GeV², about 10% deviation from λ_1 for the B_d meson.

IV. FACTORIZATION OF NONLEPTONIC DECAYS

The factorization theorem for nonleptonic B meson decays is more complicated than the semileptonic case. Nonleptonic decays involve three scales: the W boson mass M_W , at which the matching condition of the effective Hamiltonian to the full Hamiltonian is defined, the characteristic scale t , which reflects the specific dynamics of a decay mode, and the b quark transverse momentum p_\perp , which serves as a factorization scale. Below the factorization scale, dynamics is regarded as being purely nonperturbative, and absorbed into the B meson structure function, if it is soft, or into jet functions associated with light energetic final-state quarks, if it is collinear. Above the factorization scale, PQCD is reliable, and radiative corrections are absorbed into a hard b quark decay amplitude characterized by t , or into a harder function characterized by M_W . In this section we shall demonstrate how to construct the three-scale factorization theorem for nonleptonic B meson decays [11].

Consider a nonleptonic b quark decay $b \rightarrow cq\bar{q}'$ through a W boson emission up to $O(\alpha_s)$ virtual gluon corrections. The sum of the full diagrams does not possess ultraviolet divergences because of the current conservation and the presence of the W boson propagator. Reexpress a full diagram into two terms, where the first term is obtained by shrinking the W boson line into a point, and the second term is the difference of the full diagram and the first term. The first term corresponds to a local four-fermion operator $(\bar{q}q')(\bar{c}b)$ appearing in the full or effective Hamiltonian with $(\bar{q}q') = \bar{q}\gamma_\mu(1-\gamma_5)q'$ the $V-A$ current, and is absorbed into a decay amplitude $H'(p_b, p_j, \mu)$, p_j being the outgoing quark momenta. H' is characterized by momenta smaller than the W boson mass M_W , since gluons in H' do not “see” the W boson. Note that the factorization in H' , which still contains the contributions characterized by the hadronic scale, is not complete yet. The second term, characterized by momenta of order M_W due to the subtraction term, is absorbed into the harder function $H_r(M_W, \mu)$. Following the steps that lead to Eq. (16), we derive the factorization formula for the b quark decay width [11],

$$\Gamma = H_r(M_W, \mu) \times H'(p_b, p_j, \mu), \quad (33)$$

Though the full diagrams are ultraviolet finite, the factorization introduces the dependence on the renormalization scale μ into H_r and H' , because the diagrams with the W boson line shrunk are divergent.

We then investigate infrared divergences from the diagrams contained in H' . These diagrams are basically similar to those in Fig. 1, where the four-fermion vertices have been adopted, and the final-state quark may represent the c , q , or \bar{q}' quark. At this stage real gluon corrections are taken into account. If the final-state quark is c , the discussion the same as in Sec. II leads to the B meson structure function defined by Eq. (17), which is thus universal. If the final-state quark is q , the loop integrals for Figs. 1(c) and 1(d) are written as

$$(\not{p}_j + m)\Sigma'^{(c)} = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} (\not{p}_j + m) \gamma_\mu \frac{\not{p}_j - \not{l} + m}{(p_j - l)^2 - m^2} \gamma^\mu \frac{\not{p}_j + m}{p_j^2 - m^2} \frac{1}{l^2} \times \delta(p_j^2 - m^2), \quad (34)$$

$$(\not{p}_j + m)\Sigma'^{(d)} = -g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} \frac{\not{p}_j + m}{p_j^2 - m^2} \gamma_\mu (\not{p}_j - \not{l} + m) \gamma^\mu \frac{\not{p}_j + m}{p_j^2 - m^2} \times 2\pi \delta(l^2) \delta((p_j - l)^2 - m^2), \quad (35)$$

with p_j and m the final-state quark momentum and mass, respectively.

Performing the loop integrations directly, we obtain

$$\Sigma'^{(c)} = \frac{\alpha_s}{2\pi} C_F \left(\frac{1}{-\epsilon} - \frac{3}{2} \ln \frac{\mu^2}{m^2} \right) \delta(p_j^2 - m^2), \quad (36)$$

$$\Sigma'^{(d)} = -\frac{\alpha_s}{\pi} C_F \frac{(\mu^2 p_j^2)^\epsilon}{(p_j^2 - m^2)^{1+2\epsilon}}. \quad (37)$$

The terms finite as $m \rightarrow 0$ have been dropped and the ultraviolet pole has been subtracted in Eq. (36). Similarly, Eq. (37) is reexpressed, following Eq. (10), as

$$\Sigma'^{(d)} = -\frac{\alpha_s}{2\pi} C_F \frac{1}{-\epsilon} \delta(p_j^2 - m^2). \quad (38)$$

Obviously, the soft poles $1/(-\epsilon)$ disappear in the sum of Eqs. (36) and (38) as expected. For $m \neq 0$, there are no other infrared divergences, the same as in the c quark case. However, an additional infrared divergence, *i.e.*, the collinear divergence, appears when the invariant mass p_j^2 of the final-state quark is equal to m^2 and vanishes, which remains even in the sum of Eqs. (36) and (38). This divergence comes from the region with the loop momentum l being parallel to p_j and not small, for which both $(p_j - l)^2$ and l^2 approach zero. Note that p_j^2 vanishes only in the kinematic end-point region. To absorb this end-point singularity, we introduce a jet function associated with the energetic outgoing light quark. Figures 1(c) and 1(d) are then trivially factorized into the jet function.

For Figs. 1(e) and 1(f) with a gluon attaching the b quark and the q quark, the discussion is the same as for the c quark case, if the q quark is massive: there are no collinear divergences, and soft divergences cancel, as the two diagrams are convoluted with the B meson structure function. If the q quark is massless, the loop integrals for Figs. 1(e) and 1(f) are written as

$$\not{p}_j \Sigma'^{(e)} = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} \not{p}_j \gamma_\mu \frac{\not{p}_j - \not{l}}{(p_j - l)^2} \Gamma_4 \frac{\not{p}_b - \not{l} + M_b}{(p_b - l)^2 - M_b^2} \gamma^\mu \frac{1}{l^2} \delta(p_j^2), \quad (39)$$

$$\not{p}_j \Sigma'^{(f)} = -g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} \frac{\not{p}_j}{p_j^2} \gamma_\mu (\not{p}_j - \not{l}) \Gamma_4 \frac{\not{p}_b - \not{l} + M_b}{(p_b - l)^2 - M_b^2} \gamma^\mu 2\pi \delta(l^2) \delta((p_j - l)^2), \quad (40)$$

where Γ_4 represents the four-fermion vertex. Similarly, collinear divergences from l parallel to p_j with $p_j^2 \rightarrow 0$ exist and remain after summing $\Sigma'^{(e)}$ and $\Sigma'^{(f)}$. These collinear divergences should be absorbed into the jet function associated with the q quark.

The factorization of Figs. 1(e) and 1(f) in the collinear region requires an eikonal approximation. For convenience, we assume that p_j is in the plus direction at the kinematic end point. When l is parallel to p_j , only γ_- in the gamma matrices γ_μ contributes, indicating that only γ_- in γ^μ contributes, and that \not{l} in the numerator of the b quark propagator is negligible. Using the equation of motion for the b quark spinor, we have the eikonal approximation,

$$\frac{\not{p}_b + M_b}{(p_b - l)^2 - M_b^2} \gamma^\mu = \frac{2p_b^\mu}{l^2 - 2p_b \cdot l} \approx \frac{p_b^- \delta^{-\mu}}{-p_b \cdot l} = -\frac{n^\mu}{n \cdot l}. \quad (41)$$

That is, the collinear gluon decouples from the b quark, and attaches the eikonal line in the direction n defined in Sec. II. Using Eq. (41), Eqs. (39) and (40) reduce to

$$\Sigma_{\text{coll}}^{(e)} = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \gamma_\mu \frac{\not{p}_j - \not{l}}{(p_j - l)^2} \frac{n^\mu}{n \cdot l} \frac{1}{l^2} \delta(p_j^2), \quad (42)$$

$$\Sigma_{\text{coll}}^{(f)} = g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{1}{p_j^2} \gamma_\mu (\not{p}_j - \not{l}) \frac{n^\mu}{n \cdot l} 2\pi \delta(l^2) \delta((p_j - l)^2). \quad (43)$$

Since the eikonal approximation in Eq. (41) also holds for a massless quark propagator with $M_b = 0$, the factorization of the other set of diagrams with a gluon attaching the q and \bar{q}' quarks in the collinear region is similar: the collinear gluon decouples from the \bar{q}' quark and attaches the eikonal line in the direction n . Extending the above analysis to all orders, the jet function is defined as the collection of infinite many gluon exchanges among the q quark and the eikonal lines in the direction n on both sides of the final-state cut. Note that the self-energy corrections to the eikonal line are excluded. The above conclusion applies to the other final-state \bar{q}' quark: if the \bar{q}' quark is massless, there exist collinear divergences at the kinematic end point, and another jet function needs to be introduced.

With the jet functions and the B meson structure function defined in Eq. (17), all the infrared divergences in nonleptonic B meson decays are extracted and factorized appropriately. The hard b quark decay amplitude $H(t, \mu)$ is then defined as the difference of H' and those diagrams which have been absorbed into the jet functions and the B meson structure function, and thus calculable in perturbation theory. Following Eq. (16), we obtain the factorization of H' ,

$$H'(p_b, p_j, \mu) = H(t, \mu) \times \prod_j J_j(p_j, \mu) \times f(k^+, \mu), \quad (44)$$

where the index j runs over the light final-state quarks. Note that a trace of H and J_j is necessary, since the jet function carries the spin structure of the corresponding final-state quark. Combining Eqs. (33) and (44), the three-scale factorization formula for nonleptonic B meson decay widths is written as

$$\Gamma = H_r(M_W, \mu) \times H(t, \mu) \times \prod_j J_j(p_j, \mu) \times f_B(z, \mu), \quad (45)$$

where the structure function $f(k^+, \mu)$ has been transformed into the distribution function $f_B(z, \mu)$ [7]. The μ dependence will disappear after performing a RG analysis, since a decay width is scale-independent. Note that the μ dependence of the B meson distribution function for the semileptonic decays was not considered. We shall show in the next section that such a μ dependence is not necessary in our formalism.

V. LOGARITHMIC SUMMATIONS

To regularize the collinear divergences, we associate extra transverse momentum p_\perp , originating from the initial b quark, with the final-state quarks, which renders them off-shell by p_\perp . This transverse momentum can be regarded as the factorization scale, above which RG evolutions are reliable, and below which dynamics is absorbed into the B meson distribution function and the jet functions. We work in the impact parameter b space, which is the Fourier conjugate variable of p_\perp . Radiative corrections to nonleptonic decays then produce two types of large logarithms, $\ln(M_W/t)$ and $\ln(tb)$. Especially, the double logarithms $\ln^2(\bar{p}_j b)$, \bar{p}_j being the large longitudinal component of the final-state quark momentum p_j , appear in the jet functions at the kinematic end points. These logarithmic corrections should be summed to all orders using RG equations and the resummation technique [8] in order to improve perturbative expansions. The summation of the logarithms $\ln(M_W/t)$ is identified as the Wilson coefficients in the effective Hamiltonian, which describes the evolution from the characteristic scale M_W of the harder function to the characteristic scale t of the hard amplitude. The summation of the logarithms $\ln(tb)$ leads to the evolution from t to the factorization scale $1/b$.

The resummation of the double logarithms $\ln^2(\bar{p}_j b)$ for a jet with a small invariant mass is written, in the b space, as [6,12]

$$J_j(\bar{p}_j, b, \mu) = J_j(b, \mu) \exp[-2s(\bar{p}_j, b)]. \quad (46)$$

The Sudakov exponent s is given by

$$s(\bar{p}_j, b) = \int_{1/b}^{\bar{p}_j} \frac{d\mu}{\mu} \left[\ln \left(\frac{\bar{p}_j}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right], \quad (47)$$

where the factors A and B are expanded as

$$\begin{aligned} A(\alpha_s) &= A^{(1)} \frac{\alpha_s}{\pi} + A^{(2)} \left(\frac{\alpha_s}{\pi} \right)^2, \\ B(\alpha_s) &= \frac{2}{3} \frac{\alpha_s}{\pi} \ln \left(\frac{e^{2\gamma-1}}{2} \right), \end{aligned} \quad (48)$$

in order to take into account the next-to-leading-logarithm summation. We adopt the one-loop running coupling constant,

$$\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_1 \ln(\mu^2/\Lambda^2)}. \quad (49)$$

The above coefficients $A^{(1)}$, $A^{(2)}$ and β_1 are

$$\beta_1 = \frac{33 - 2n_f}{12}, \quad A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln \left(\frac{e^\gamma}{2} \right), \quad (50)$$

where $n_f = 5$ is the number of quark flavors, and γ the Euler constant. The Sudakov factor $\exp(-2s)$ exhibits strong suppression at large b , and approaches unity as $\bar{p}_j < 1/b$. In this region the final-state quarks are regarded as being highly off-shell, and absorbed into the hard amplitude. Hence, double logarithms do not exist, *i.e.*, $\exp(-2s) \rightarrow 1$, and it is not necessary to introduce a jet function. For the detailed derivation of Eq. (46), refer to [12,13]. The large scale \bar{p}_j will be chosen as the sum of the longitudinal components of p_j , *i.e.*, $\bar{p}_j = p_j^+ + p_j^-$ in the factorization formulas presented in the next section.

The initial condition $J_j(b, \mu)$ of the resummation still contains single logarithms $\ln(b\mu)$, which are summed by RG equations. The RG solution of the jet function is given by

$$J_j(b, \mu) = J_j^{(0)} \exp \left[- \int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_j(\alpha_s(\bar{\mu})) \right], \quad (51)$$

with the anomalous dimension γ_j . It is more convenient to compute γ_j in the axial gauge $n \cdot A = 0$, since the eikonal lines in the direction n disappear. The n dependence goes into the gluon propagator $(-i/l^2)N^{\mu\nu}(l)$, with

$$N^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu l^\nu + n^\nu l^\mu}{n \cdot l}. \quad (52)$$

The lowest-order self-energy correction to the final-state quark is written as

$$\Sigma_q = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \gamma_\mu \frac{\not{p}_j - \not{l}}{(p_j - l)^2} \gamma_\nu \frac{N^{\mu\nu}}{l^2} \frac{\not{p}_j}{p_j^2}, \quad (53)$$

which is obtained by replacing the gluon propagator $-ig^{\mu\nu}/l^2$ in Eq. (34) by $-iN^{\mu\nu}/l^2$. Extracting the ultraviolet poles from the above loop integral, we find that the $g^{\mu\nu}$ term and the second term in Eq. (52) give $-\alpha_s C_F/(4\pi) \times (1/\epsilon)$ and $\alpha_s C_F/\pi \times (1/\epsilon)$, respectively. Their sum, $\Sigma_q = \alpha_s/\pi \times (1/\epsilon)$, leads to the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$ in the axial gauge. γ_j is twice of γ_q , *i.e.*, $\gamma_j = 2\gamma_q$, because the self-energy corrections occur before and after the final-state cut. The initial condition $J_j^{(0)}$, with the large logarithms collected by the Sudakov factor and by the RG evolution, can be approximated by its tree-level expression, *i.e.*, the final-state cut.

The RG solution of the B meson distribution function is

$$f_B(z, \mu) = f_B(z) \exp \left[- \int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_S(\alpha_s(\bar{\mu})) \right]. \quad (54)$$

where the initial condition $f_B(z)$ absorbs the nonperturbative dynamics below the scale $1/b$. The anomalous dimension γ_S is also computed in the axial gauge, under which the path-ordered exponential in Eq. (17) is

equal to unity. Hence, we need to consider only the self-energy correction to the h_v field in Fig. 1(a). The loop integral is written as

$$\Sigma_v = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} \frac{v_\mu}{v \cdot k} \frac{v_\nu}{v \cdot (k-l)} \frac{N^{\mu\nu}}{l^2}, \quad (55)$$

where the residual momentum k approaches zero. Using the expansion

$$\frac{1}{v \cdot (k-l)} = -\frac{1}{v \cdot l} - \frac{v \cdot k}{(v \cdot l)^2}, \quad (56)$$

Eq. (55) reduces to

$$\Sigma_v = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} \frac{v_\mu v_\nu}{(v \cdot l)^2} \frac{N^{\mu\nu}}{l^2}, \quad (57)$$

which is the same as $\Sigma_{\text{soft}}^{(a)}$ in Eq. (4) but with the gluon propagator $-ig^{\mu\nu}/l^2$ replaced by $-iN^{\mu\nu}/l^2$. Therefore, Eq. (17) indeed generates the factor $1 + \Sigma_{\text{soft}}^{(a)}$ in Eq. (16) at first order. It is easy to find that the contribution from the second term in Eq. (52) vanishes, and that the $g^{\mu\nu}$ term gives $\Sigma_v = \alpha_s C_F / (2\pi) \times (1/\epsilon)$. Adding the self-energy corrections on both sides of the final-state cut, we derive the anomalous dimension $\gamma_S = -\alpha_s C_F / \pi$ of the B meson distribution function.

The RG solution of H_r is

$$H_r(M_W, \mu) = H_r(M_W, M_W) \exp \left[\int_\mu^{M_W} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{H_r}(\alpha_s(\bar{\mu})) \right], \quad (58)$$

with γ_{H_r} the anomalous dimension of H_r . The initial condition $H_r(M_W, M_W)$ can be safely approximated by its lowest-order expression $H_r^{(0)} = 1$, since the large logarithms $\ln(M_W/\mu)$ have been organized into the exponential, which is identified as the Wilson coefficient $c(\mu)$,

$$c(\mu) \equiv \exp \left[\int_\mu^{M_W} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{H_r}(\alpha_s(\bar{\mu})) \right]. \quad (59)$$

For the explicit expressions of the Wilson coefficients, refer to [14].

At last, the anomalous dimension of the hard amplitude H is given by $\gamma_H = -\gamma_{H_r} - \sum_j \gamma_j - \gamma_S$ because of the scale invariance of a decay width in the full theory. Applying the RG analysis, we obtain

$$H(t, \mu) = H(t, t) \exp \left\{ - \int_\mu^t \frac{d\bar{\mu}}{\bar{\mu}} \left[\gamma_{H_r}(\alpha_s(\bar{\mu})) + \sum_j \gamma_j(\alpha_s(\bar{\mu})) + \gamma_S(\alpha_s(\bar{\mu})) \right] \right\}. \quad (60)$$

The scale t will be chosen as the maximal relevant scales,

$$t = \max \left(\bar{p}_j, \frac{1}{b} \right). \quad (61)$$

Since the large b region is Sudakov suppressed by $\exp(-2s)$ as stated before, that is, t remains as a hard scale, and the large logarithms $\ln(t/\mu)$ in H have been grouped into the exponential, the initial condition $H(t, t)$ is calculable in perturbation theory. Hence, the Sudakov factor, though important only in the end-point region, improves the applicability of PQCD to inclusive nonleptonic heavy hadron decays.

Substituting Eqs. (46), (51), (54), (58) and (60) into Eq. (45), we derive the RG improved factorization formula for the nonleptonic B meson decay widths,

$$\Gamma = c(t) H(t, t) f_B(z) \exp \left\{ - \sum_j \left[2s(\bar{p}_j, b) + \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_j(\alpha_s(\bar{\mu})) \right] - \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_S(\alpha_s(\bar{\mu})) \right\}, \quad (62)$$

where the cancellation of the μ dependences among the convolution factors is explicit, and the two-stage evolutions from $1/b$ to t and from t to M_W have been established. Note that Eq. (62) in fact denotes the

integrand appearing in the factorization formula, and that the variables t and b will be integrated out. The initial conditions $J_j^{(0)}$ in Eq. (51) have been incorporated into the evaluation of $H(t, t)$. We emphasize that the Wilson coefficient appears as a convolution factor in the three-scale factorization theorem, instead of a constant (once its argument μ is set to a common value for all decay modes) in the HQET approach. Since the perturbative factors depend on the hadron kinematics, they vary in different decay modes of a hadron and in different hadrons. It will be shown that these perturbative effects play an essential role in the explanation of the b -hadron lifetimes.

In the semileptonic case there are not the Wilson coefficient $c(t)$ and the Sudakov factor $\exp(-2s)$ because of the absence of the double logarithms. Therefore, the large scales \bar{p}_j do not exist, and t is equal to $1/b$, for which the RG evolution factors governed by the anomalous dimensions γ_j and γ_S disappear. This is the reason we did not perform the RG analysis for the semileptonic decays in Sec. II.

VI. FACTORIZATION FORMULAS AND NUMERICAL ANALYSIS

After developing the factorization theorems, we present the factorization formulas for the B meson semileptonic and nonleptonic decay widths. Define the momentum of the light degrees of freedom as $p = (p^+, 0, \mathbf{p}_\perp)$. The B meson is at rest with the momentum $P_B = M_B/\sqrt{2}(1, 1, \mathbf{0})$. The b quark momentum is then written as $p_b = P_B - p = (z M_B/\sqrt{2}, M_B/\sqrt{2}, -\mathbf{p}_\perp)$, where the momentum fraction $z \equiv p_b^+/P_B^+ = 1 - \sqrt{2}p^+/M_B$ is the same as that defined in Sec. III. The lepton and neutrino momenta involved in the semileptonic decays $B(P_B) \rightarrow X_c + l(p_l) + \bar{\nu}(p_\nu)$ are expressed, in terms of light-cone coordinates, as

$$p_l = (p_l^+, p_l^-, 0_\perp), \quad p_\nu = (p_\nu^+, p_\nu^-, \mathbf{p}_{\nu\perp}), \quad (63)$$

where the minus component p_l^- vanishes for a massless lepton. For convenience, we adopt the scaling variables,

$$x = \frac{2E_l}{M_B}, \quad y = \frac{q^2}{M_B^2}, \quad y_0 = \frac{2q_0}{M_B}, \quad (64)$$

with the kinematic ranges,

$$\begin{aligned} 2\sqrt{\alpha} \leq x \leq 1 + \alpha - \beta, \\ \alpha \leq y \leq \alpha + (1 + \alpha - \beta - x) \frac{x + \sqrt{x^2 - 4\alpha}}{2 - x - \sqrt{x^2 - 4\alpha}}, \\ x + \frac{2(y - \alpha)}{x + \sqrt{x^2 - 4\alpha}} \leq y_0 \leq 1 + y - \beta, \end{aligned} \quad (65)$$

where E_l is the lepton energy and $q \equiv p_l + p_\nu$ the lepton pair momentum. The constants α and β are

$$\alpha \equiv \frac{M_l^2}{M_B^2}, \quad \beta \equiv \frac{M_D^2}{M_B^2}, \quad (66)$$

M_l and M_D being the lepton mass and the D meson mass, respectively. M_D arises as the minimal invariant mass of the decay product X_c . For the derivation of Eq. (65), refer to the Appendix.

The factorization formula for the semileptonic decay width is given, in the b space, by [15]

$$\frac{\Gamma_{\text{SL}}}{\Gamma_0} = \frac{M_B^2}{2\pi} \int dx dy dy_0 \int_{z_{\min}}^1 dz \int_0^\infty db b f_B(z) \tilde{J}_c(x, y, y_0, z, b) H(x, y, y_0, z), \quad (67)$$

with $\Gamma_0 \equiv (G_F^2/16\pi^3)|V_{cb}|^2 M_B^5$. The momentum fraction z approaches 1 as the b quark carries the whole B meson momentum in the plus direction. The minimum of z , determined by the condition $p_c^2 > M_c^2$, is

$$z_{\min} = \frac{\frac{y_0}{2} - y + \frac{M_c^2}{M_B^2} - \frac{x}{\sqrt{x^2 - 4\alpha}} \left[-\frac{y_0}{2} + \frac{y}{x} + \frac{\alpha}{x} \right]}{1 - \frac{y_0}{2} - \frac{x}{\sqrt{x^2 - 4\alpha}} \left[-\frac{y_0}{2} + \frac{y}{x} + \frac{\alpha}{x} \right]}. \quad (68)$$

The function \tilde{J}_c denotes the Fourier transformation of the final-state cut on the c quark line, which is in fact part of the hard amplitude. J_c and the lowest-order H in momentum space are [15]

$$\begin{aligned} J_c &= \delta(p_c^2 - M_c^2), \\ &= \delta \left(M_B^2 \left\{ z - (1+z) \frac{y_0}{2} + y + \frac{x(1-z)}{\sqrt{x^2 - 4\alpha}} \left[-\frac{y_0}{2} + \frac{y}{x} + \frac{\alpha}{x} \right] \right\} - M_c^2 - \mathbf{p}_\perp^2 \right), \end{aligned} \quad (69)$$

$$\begin{aligned} H &= (p_b \cdot p_\nu)(p_l \cdot p_c), \\ &= \left((y_0 - x) \left\{ 1 - \frac{(1-z)}{2} \left(1 - \frac{x}{\sqrt{x^2 - 4\alpha}} \right) \right\} - \frac{(1-z)}{\sqrt{x^2 - 4\alpha}} (y - \alpha) \right) \\ &\quad \times \left(\frac{x}{2} \left\{ 1 + z + (1-z) \frac{\sqrt{x^2 - 4\alpha}}{x} \right\} - y - \alpha \right). \end{aligned} \quad (70)$$

The universal B meson distribution function f_B , determined from the $B \rightarrow X_s \gamma$ decay, has been given in Eq. (27), which minimizes the model dependence of our predictions.

For the nonleptonic decays, we consider the modes $b \rightarrow c\bar{c}s$ and $b \rightarrow c\bar{u}d$. Ignoring the penguin operators, the effective Hamiltonian for the $b \rightarrow c\bar{c}s$ decay is

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^* [c_1(\mu) O_1(\mu) + c_2(\mu) O_2(\mu)], \quad (71)$$

with G_F the Fermi coupling constant, V 's the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements, the four-quark operators $O_1 = (\bar{s}b)(\bar{c}c)$ and $O_2 = (\bar{c}b)(\bar{s}c)$, and the initial conditions $c_1(M_W) = 1$ and $c_2(M_W) = 0$. For the $b \rightarrow c\bar{u}d$ decay, V_{cs} and the \bar{c} and s quark fields are replaced by V_{ud} and the \bar{u} and d quark fields, respectively. It is simpler to work with the operators $O_\pm = \frac{1}{2}(O_2 \pm O_1)$ and their corresponding coefficients $c_\pm(\mu) = c_2(\mu) \pm c_1(\mu)$, since they are multiplicatively renormalized. In the leading logarithmic approximation c_\pm are given by [14]

$$c_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\frac{-6\gamma_\pm}{33-2n_f}}, \quad (72)$$

with the constants $2\gamma_+ = -\gamma_- = -2$ and $n_f = 5$.

To simplify the analysis, we route the transverse momentum p_\perp of the b quark through the outgoing c quark as in the semileptonic case, such that the c quark momentum is the same as before. For kinematics, we make the correspondence with the \bar{c} (\bar{u}) quark carrying the momentum of the massive (light) lepton τ (e and μ) and with the s and d quarks carrying the momentum of $\bar{\nu}$. The scaling variables are then defined exactly by Eq. (64). The factorization formula for the nonleptonic decay widths is written, according to the three-scale factorization theorem in Sec. IV, as [15]

$$\begin{aligned} \frac{\Gamma_{\text{NL}}}{\Gamma_0} &= \frac{M_B^2}{2\pi} \int dx dy dy_0 \int_{z_{\text{min}}}^1 dz \int_0^\infty b db \left[\frac{N_c+1}{2} c_+^2(t) + \frac{N_c-1}{2} c_-^2(t) \right] \\ &\quad \times f_B(z) \tilde{J}_c(x, y, y_0, z, b) H(x, y, y_0, z) S(\bar{p}_j, t, b). \end{aligned} \quad (73)$$

The B meson distribution function f_B , being universal, is the same as that for the semileptonic decays. The factor S is the result of the Sudakov resummation and the RG evolutions, given by

$$S(\bar{p}_j, t, b) = \exp \left\{ - \sum_j \left[2s(\bar{p}_j, b) + \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_j(\alpha_s(\bar{\mu})) \right] - \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_S(\alpha_s(\bar{\mu})) \right\}, \quad (74)$$

with $j = s$ for the $b \rightarrow c\bar{c}s$ mode and $j = \bar{u}, d$ for the $b \rightarrow c\bar{u}d$ mode. The explicit expressions of \bar{p}_j are

$$\bar{p}_s = \bar{p}_d = \frac{M_B}{\sqrt{2}} (y_0 - x), \quad \bar{p}_u = \frac{x M_B}{\sqrt{2}}. \quad (75)$$

The anomalous dimensions $\gamma_j = -2\alpha_s/\pi$ and $\gamma_S = -C_F\alpha_s/\pi$ have been computed in Sec. V. It has been found that the single-logarithm evolutions governed by γ_j and γ_S enhance the nonleptonic branching ratios and thus lower the semileptonic branching ratios [15], consistent with the observation in [16].

The above formalism can be generalized to the B_s meson and Λ_b baryon decays straightforwardly, for which the functional forms of the hard amplitudes H , the final-state cut J_c , the Wilson coefficients c_{\pm} , and the Sudakov evolution factor S remain the same, since they are evaluated at the quark level. The only differences arise from the replacement of the B_d meson mass M_B by the B_s meson mass M_{B_s} and by the Λ_b baryon mass M_{Λ_b} , and from the heavy hadron distribution functions given in Eqs. (30) and (31). We then proceed with the numerical analysis of the factorization formulas in Eqs. (67) and (73) for the semileptonic and nonleptonic b -hadron decays, respectively, choosing the CKM matrix elements $|V_{cs}| = |V_{ud}| = 1.0$ and $|V_{cb}| = 0.044$, and the masses $M_c = 1.6$ GeV, $M_D = 1.869$ GeV and $M_\tau = 1.7771$ GeV. Our predictions are not sensitive to the change of M_c : the difference of the results for $M_c = 1.5$ GeV from those for $M_c = 1.6$ GeV is less than 5%. We obtain the lifetimes $\tau(B_d) = 1.56$ ps, $\tau(B_s) = 1.46$ ps and $\tau(\Lambda_b) = 1.22$ ps, or the ratios $\tau(B_s)/\tau(B_d) = 0.94$ and $\tau(\Lambda_b)/\tau(B_d) = 0.78$.

The experimental data of the b -hadron lifetimes are summarized below. The lifetimes of the B_d and B_s mesons and of the Λ_b baryon are $\tau(B_d) = (1.55 \pm 0.03)$ ps, $\tau(B_s) = (1.47 \pm 0.06)$ ps and $\tau(\Lambda_b) = (1.23 \pm 0.05)$ ps, respectively, from the CERN e^+e^- collider LEP measurements [1]. Recent CDF results yield $\tau(\Lambda_b) = (1.32 \pm 0.16)$ ps [17]. The lifetime ratios are then $\tau(B_s)/\tau(B_d) = 0.95 \pm 0.06$ and $\tau(\Lambda_b)/\tau(B_d) = 0.79 \pm 0.06$ from LEP, and $\tau(\Lambda_b)/\tau(B_d) = 0.85 \pm 0.11$ from CDF. Obviously, our predictions are almost the same as the central values of the LEP data. Employing the B_u meson distribution function in Eq. (32), we obtain $\tau(B_u) = 1.62$ ps or the ratio $\tau(B_u)/\tau(B_d) = 1.04$, which is in agreement with the experimental data $\tau(B_u) = 1.65 \pm 0.04$ ps in [10].

To test the sensitivity of our predictions to the variation of λ_{1B_s} and $\lambda_{1\Lambda_b}$, we adopt the HQET parameters $\lambda_1^{\text{meson}} \sim \lambda_1^{\text{baryon}} = -0.4$ GeV² from QCD sum rules for λ_{1B_s} and $\lambda_{1\Lambda_b}$ (with the correspondiong $\bar{\Lambda}_{B_s} = 0.77$ GeV and $\bar{\Lambda}_{\Lambda_b} = 1.03$ GeV). The lifetimes $\tau(B_s) = 1.49$ ps and $\tau(\Lambda_b) = 1.26$, or the ratios $\tau(B_s)/\tau(B_d) = 0.96$ and $\tau(\Lambda_b)/\tau(B_d) = 0.81$, are derived, which are still much smaller than those from the HQET approach. This test indicates that our results are insensitive to the variation of the distribution functions (less than 4% under 40% variation of the HQET parameters), and that the explanation of the experimental data is mainly due to the PQCD effects. To confirm this observation, we set all the b -hadron masses in the phase space factors Γ_0 and in the perturbative factors to the b quark mass M_b in Eqs. (67) and (73). That is, the difference among the factorization formulas for b -hadron decays resides only in the distribution functions. The lifetime ratios $\tau(B_s)/\tau(B_d) = 1.01$ and $\tau(\Lambda_b)/\tau(B_d) = 1.09$, which are even greater than unity, are obtained. Therefore, the perturbative contributions are indeed responsible for the low b -hadron lifetime ratios.

The above results are aummarized in Table I. In Table I we also present the branching ratio of each mode in the B_d and B_s meson decays and in the Λ_b baryon decays in terms of the quantities $r_{\tau\nu} = B(b \rightarrow c\tau\bar{\nu})/B(b \rightarrow cl\bar{\nu})$, $r_{ud} = B(b \rightarrow c\bar{u}d)/B(b \rightarrow cl\bar{\nu})$, $r_{cs} = BR(b \rightarrow c\bar{c}s)/B(b \rightarrow cl\bar{\nu})$, the semileptonic branching ratio B_{SL} and the average charm yield $\langle n_c \rangle$ per b -hadron decay. It is observed that our predictions of $B_{\text{SL}} = 10.16\%$ and $\langle n_c \rangle = 1.17$ for the B meson are consistent with the experimental data: $B_{\text{SL}} = (10.19 \pm 0.37)$ and $\langle n_c \rangle = (1.12 \pm 0.05)$ from the CLEO group [18], and $B_{\text{SL}} = (11.12 \pm 0.20)$ and $\langle n_c \rangle = (1.20 \pm 0.07)$ from the LEP measurements [19]. It is also observed that B_{SL} , $\langle n_c \rangle$ and all r 's except r_{ud} increase a bit with the b -hadron masses. It is interesting to examine these tendencies in future experiments.

VII. CONCLUSION

In this paper we have developed the PQCD factorization theorems for inclusive b -hadron decays by carefully analyzing the infrared divergences in radiative corrections to b quark decays. Radiative corrections characterized by the hadronic scale are absorbed into the heavy hadron distribution function or into the jet functions. Above the hadronic scale, perturbative contributions characterized by the b -hadron mass and the W boson mass are absorbed into the hard b quark decay amplitude and the harder function, respectively. Various large logarithmic corrections have been summed to all orders using the resummation technique and the RG equations, leading to the evolution factors among the three characteristic scales, such that the factorization formulas are μ -independent. We have shown that the soft cancellation between virtual and real corrections demands the introduction of the heavy hadron distribution function, and thus the b -hadron kinematics into our formalism. Using the PQCD factorization theorems, we have been able to explain the lifetimes of the b -hadrons B_d , B_u , B_s and Λ_b . It has been confirmed that our predictions are insensitive to the variation of the distribution functions as expected from the quark-hadron duality, and that the low b -hadron lifetime ratios are mainly attributed to the perturbative contributions.

I thank Profs. H.Y. Cheng and H.L. Yu for useful discussions. This work was supported by the National

APPENDIX

In this appendix we derive the phase space constraints for the decay $b \rightarrow cl\bar{\nu}$. The constraints for the decay $b \rightarrow c\bar{c}s$ are the same with the \bar{c} and s quarks corresponding to the l and $\bar{\nu}$ leptons, respectively. The constraints for the decay $b \rightarrow c\bar{u}d$ is the massless lepton case of the results presented below.

We derive the constraint of $x = 2E_l/M_H$ from the energy and momentum conservations associated with the decay $H \rightarrow X_c l \bar{\nu}$,

$$\begin{aligned} M_H &= E_X + E_l + E_\nu , \\ 0 &= \mathbf{p}_X + \mathbf{p}_l + \mathbf{p}_\nu . \end{aligned} \quad (76)$$

Because of $E_l^2 = |\mathbf{p}_l|^2 + M_l^2$, the minimum of E_l occurs at the minimum of $|\mathbf{p}_l|$, $|\mathbf{p}_l| = 0$, and thus the minimum of x is given by

$$x_{\min} = \frac{2M_l}{M_H} . \quad (77)$$

To obtain the maximum of x , we find the maximum of $|\mathbf{p}_l|$ under Eq. (76),

$$M_H = \sqrt{(\mathbf{p}_l + \mathbf{p}_\nu)^2 + M_X^2} + \sqrt{|\mathbf{p}_l|^2 + M_l^2 + |\mathbf{p}_\nu|^2} , \quad (78)$$

into which $\mathbf{p}_X = -(\mathbf{p}_\tau + \mathbf{p}_\nu)$ has been inserted. It is easy to observe that the maximum of $|\mathbf{p}_l|$ corresponds to $\mathbf{p}_\nu = 0$, leading to

$$|\mathbf{p}_{l \max}|^2 = \frac{(M_H^2 + M_l^2 - M_X^2)^2 - 4M_H^2 M_l^2}{4M_H^2} , \quad (79)$$

and

$$x_{\max} = 1 + \frac{M_l^2}{M_H^2} - \frac{M_X^2}{M_H^2} . \quad (80)$$

Combining Eqs. (77) and (80), we obtain

$$\frac{2M_l}{M_H} \leq x \leq 1 + \frac{M_l^2}{M_H^2} - \frac{M_D^2}{M_H^2} , \quad (81)$$

where M_X has been replaced by the D meson mass M_D , the mass of the lightest charmed hadron.

We then derive the constraint of the kinematic variable $y = q^2/M_H^2$ with

$$q^2 = (p_l + p_\nu)^2 = M_l^2 + 2(E_l - |\mathbf{p}_l| \cos \theta)E_\nu , \quad (82)$$

where θ is the angle between the vectors \mathbf{p}_l and \mathbf{p}_ν . The minimum of q^2 occurs, as $\cos \theta = 1$ and E_ν takes its minimal value under the constraint from Eq. (76),

$$M_H - E_l = \sqrt{|\mathbf{p}_l|^2 + E_\nu^2 + 2|\mathbf{p}_l|E_\nu \cos \theta + M_X^2} + E_\nu . \quad (83)$$

Obviously, the minimum of E_ν is zero, which can be achieved by increasing M_X . We then have

$$(q^2)_{\min} = M_l^2 . \quad (84)$$

On the other hand, the maximum of q^2 occurs, as $\cos \theta = -1$ and E_ν takes its maximal value under Eq. (83), which corresponds to the minimum of M_X . Setting $M_X = M_D$, we obtain

$$E_{\nu \max} = \frac{M_H^2 + M_l^2 - M_D^2 - 2M_H E_l}{2(M_H - E_l - \sqrt{E_l^2 - M_l^2})} , \quad (85)$$

and

$$(q^2)_{\max} = M_l^2 + (M_H^2 + M_l^2 - M_D^2 - 2M_H E_l) \frac{E_l + \sqrt{E_l^2 - M_l^2}}{M_H - E_l - \sqrt{E_l^2 - M_l^2}}. \quad (86)$$

Combining Eqs. (84) and (86), we have

$$\frac{M_l^2}{M_H^2} \leq y \leq \frac{M_l^2}{M_H^2} \left(1 + \frac{M_l^2}{M_H^2} - \frac{M_D^2}{M_H^2} - x \right) \left(\frac{2}{2 - x - \sqrt{x^2 - 4M_l^2/M_H^2}} - 1 \right). \quad (87)$$

At last, we derive the range of the variable $y_0 = 2q_0/M_H$ with

$$y_0 = E_l + E_\nu, \quad (88)$$

or equivalently, the range of E_ν under the constraints of Eqs. (82) and (83). Since E_ν decreases with $\cos\theta$ in order that q^2 maintains constant, the minimum of E_ν occurs at $\cos\theta = -1$. Equation (82) then gives

$$E_{\nu \min} = \frac{q^2 - M_l^2}{2(E_l + \sqrt{E_l^2 - M_l^2})}, \quad (89)$$

and

$$y_{0 \min} = x + \frac{2(y - M_l^2/M_H^2)}{x + \sqrt{x^2 - 4M_l^2/M_H^2}}. \quad (90)$$

It is easy to observe that E_ν increases as M_X decreases. Setting $M_X = M_D$, and substituting the expression of $\cos\theta$ from Eq. (82) into (83), we obtain

$$E_{\nu \max} = \frac{M_H^2 - 2M_H E_l - M_D^2 + q^2}{2M_H}, \quad (91)$$

and

$$y_{0 \max} = 1 + y - \frac{M_D^2}{M_H^2}. \quad (92)$$

Combining Eqs. (90) and (92), we derive

$$x + \frac{2(y - M_l^2/M_H^2)}{x + \sqrt{x^2 - 4M_l^2/M_H^2}} \leq y_0 \leq 1 + y - \frac{M_D^2}{M_H^2}. \quad (93)$$

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Table I. The b -hadron lifetimes $\tau(B_d)$, $\tau(B_u)$, $\tau(B_s)$, and $\tau(\Lambda_b)$, and their ratios to $\tau(B_d)$. Predictions of $r_{\tau\nu}$, r_{ud} , r_{cs} , B_{SL} , and $\langle n_c \rangle$ for the b -hadron decays are also listed.

	$r_{\tau\nu}$	r_{ud}	r_{cs}	B_{SL}	$\langle n_c \rangle$	τ (ps)	$\tau/\tau(B_d)$
$\lambda_1 = -0.71 \text{ GeV}^2$	0.224	5.98	1.64	10.16	1.17	1.56	1.0
$\lambda_{1B_u} = -0.81 \text{ GeV}^2$	0.222	6.11	1.54	10.14	1.16	1.62	1.04
$\lambda_{1B_s} = -0.4 \text{ GeV}^2$	0.231	5.25	1.44	11.21	1.16	1.49	0.96
$\lambda_{1B_s} = -0.71 \text{ GeV}^2$	0.237	5.30	1.69	10.85	1.18	1.46	0.94
$\lambda_{1\Lambda_b} = -0.4 \text{ GeV}^2$	0.254	5.14	1.75	10.94	1.19	1.26	0.81
$\lambda_{1\Lambda_b} = -0.71 \text{ GeV}^2$	0.261	5.26	1.68	10.87	1.18	1.22	0.78

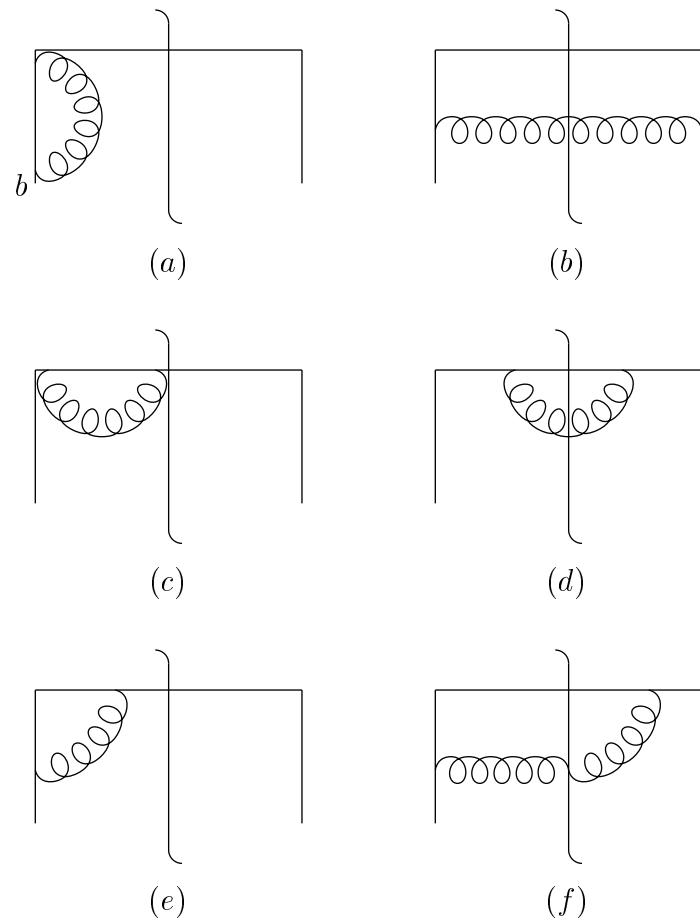


FIG. 1